MMM		HHH HHH HHH HHH HHH HHH HHH HHH HHH HH	RRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRRR		LLL LLL LLL LLL LLL LLL LLL LLL LLL LL
MMM MMM	††† †††	HHH HHH HHH HHH	RRR RRR	111 111 111	

000000 00 00 00 00	TTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTTT	\$	PPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPPP	000000 00 00 00 00	DDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDDD
		\$			

2-0

- DOUBLE PRECISION ** DOUBLE PRECISION p 16-SEP-1984 01:57:20 VAX/VMS Macro V04-00 OTS\$POWDD Table of contents Page HISTORY ; Detailed current edit history DECLARATIONS OTS\$POWDD - DOUBLE to DOUBLE giving DOUBLE result (2) (2) (4)

.TITLE OTS\$POWDD - DOUBLE PRECISION ** DOUBLE PRECISION power routine .IDENT /2-007/ ; File: OTSPOWDD.MAR Edit: JCW2007

2-0

COPYRIGHT (c) 1978, 1980, 1982, 1984 BY DIGITAL EQUIPMENT CORPORATION, MAYNARD, MASSACHUSETTS. ALL RIGHTS RESERVED.

THIS SOFTWARE IS FURNISHED UNDER A LICENSE AND MAY BE USED AND COPIED ONLY IN ACCORDANCE WITH THE TERMS OF SUCH LICENSE AND WITH THE INCLUSION OF THE ABOVE COPYRIGHT NOTICE. THIS SOFTWARE OR ANY OTHER COPIES THEREOF MAY NOT BE PROVIDED OR OTHERWISE MADE AVAILABLE TO ANW OTHER PERSON. NO TITLE TO AND OWNERSHIP OF THE SOFTWARE IS HEREBY TRANSFERRED.

THE INFORMATION IN THIS SOFTWARE IS SUBJECT TO CHANGE WITHOUT NOTICE AND SHOULD NOT BE CONSTRUED AS A COMMITMENT BY DIGITAL EQUIPMENT CORPORATION.

DIGITAL ASSUMES NO RESPONSIBILITY FOR THE USE OR RELIABILITY OF ITS SOFTWARE ON EQUIPMENT WHICH IS NOT SUPPLIED BY DIGITAL.

; FACILITY: Language support library - user callable ABSTRACT:

- Double base to double power. Floating base to double power. Double base to floating power.

Floating overflow can occur Undefined exponentiation can occur if:

1) Negative base

0 base and power is 0 or negative.

VERSION: 2

HISTORY:

AUTHOR:

Bob Hanek, 3-Mar-83: Version 2

MODIFIED BY:

2-0

```
.SBTTL HISTORY
                                                                                                                                                                                                           ; Detailed current edit history
                                     ; Edit history for Version 2 of OTS$POWDD
2-001 Implemented new algorithm. RNH 3-Mar-83
2-002 The code would overflow when underflow was the proper result. This error arouse from the ADDF3 #^X4DCO, R2, R6 code, which rounded the result so that when SUBF #^X4DCO, R6 was performed the value in R6 was greater than R2, causing R0 to be positive when it should have been negative. The bugfix was to change the test for overflow from R0 to R6 (both of these should be of the same sign). JCW 13-MAY-83.
2-003 Change INDEX table to be local instead of global. Also change the instruction format from INDEX(Rx) to INDEX[Rx] to avoid [inker errors. LEB 25-May-1983
2-004 Change remaining table references to use [Rx] instead of (Rx).
                                                           2-004 Change remaining table references to use [Rx] instead of (Rx).

Make A1 TABLE and A2_TABLE local symbols instead of globals.

LEB 26-May-1983

2-005 Change leftover table reference to use [Rx] instead of (Rx).

LEB 29-May-1983
                                                          2-006 Added two ROTL #-3,Rx,Rx instructions to scale the value of Rx back from 'index*2^3' to 'index' before A_TABLE[Rx] is referenced. The INDEX was not scaled back to yield values of 'index' instead of 'index*2^3' because the mathematics of the code used does need the value of index*2^3 in several computations. JCW 7-Jun-1983
2-007 Corrected a bug involving a SYS_F_FLTOVF_F error during a MULD R6, R2. Code was added to see if a MTH overflow message or a zero should be returned. JCW 19-Jan-1984
```

```
- DOUBLE PRECISION ** DOUBLE PRECISION p 16-SEP-1984 01:57:20 VAX/VMS Macro V04-00 DECLARATIONS 6-SEP-1984 11:28:03 [MTHRTL.SRCJOTSPOWDD.MAR;1
                                                                                                                                                                                                             (2)
                                                           .SBTTL DECLARATIONS
                                  INCLUDE FILES:
                    EXTERNAL SYMBOLS:
                                                                         GBL
MTH$$SIGNAL
MTH$K_FLOOVEMAT
MTH$K_FLOUNDMAT
MTH$K_UNDEXP
                                                          .DSABL
.EXTRN
.EXTRN
.EXTRN
.EXTRN
                                                                                                                          ; Math error routine
; floating overflow code
; Floating underflow code
; Undefined exponentiation code
                                              MACROS:
                                                          SSFDEF
                                                                                                                          : Define stack frame symbols
                                              EQUATED SYMBOLS:
00000004
00000000
00000008
000007FC
                                                         base = 4 ; base input formal - by-value exp = 12 ; exponent input formal - by-value fexp = 8 ; exponent when base is floating ACMASK = ^M< R2, R3, R4, R5, R6, R7, R8, R9, R10> ; register saving mask
```

\$AE _OI

OTS Syn

In Con Pas Syn Pas Syn

```
- DOUBLE PRECISION ** DOUBLE PRECISION p 16-SEP-1984 01:57:20 VAX/VMS Macro V04-00 DECLARATIONS 6-SEP-1984 11:28:03 [MTHRTL.SRC]OTSPOWDD.MAR;1
                                                                                                                                                                                                                                  PSECT DECLARATIONS:
                                                                                                                                   00000000
0000
0000
0000
                                                                                                                                                                                                                                                                                          .PSECT _OTS$CODE
                                                                                                                                                                                                                                                                                                                                                                                                                                              PIC,SHR,QUAD,EXE,NOWRT ; program section for OTS$ code
                                                                                                                                                                                                                                  CONSTANTS:
                                                                                                                                                                                                          The INDEX table gives the appropriate byte offset into the A1 and A2 Tables.
                                                                                                                                                              0000
0008
0010
0018
0020
0028
0030
0038
0040
0058
0068
0070
0080
0080
                                                                                                                                                                                                                                     INDEX:
                   01008033448086677780
                                      0112333445556687780
                                                                           0112333445566687780
                                                                                              0008008088088088
0112334455566777
                                                                                                                  001808008080808088
455566777
                                                                                                                                    00880808080808088
80808080808088
                                                                                                                                                                                                                                                                                       *X10

*X18

*X28

*X28

*X30

*X48

*X50

*X50

*X68

*X78

*X78
                                                                                                                                                                                                                                                                                                                                                                                                                    *X10,

*X20,

*X38,

*X40,

*X58,

*X60,

*X60,

*X70,

*X78,

*X80,
12233344558080080
                                                                                                                                                                                                                                                                                                                                          *X10,

*X228,

*X38,

*X48,

*X58,

*X58,

*X68,

*X68,

*X78,

*X80,
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              100,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1208,
1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      *X40
*X48
*X50
*X58
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      *X60
*X68
*X70
*X70
                                                                                                                                                                                                                                                                                         .ALIGN QUAD
                                                                                                                                                               For k=0,1,\ldots,16, the k-th entry of the A1 table is 2^{*}(k/16) rounded to 56 bits and the k-th entry of the A2 table 2^{*}(k/16) - A1_TABLE(k) rounded to 56 bits.
                                    00000000 00004080
487867CC AAC34085
8BD7E3EA 95C14088
11C373AB C3D34091
B8A9518D 37F04098
A1126091 F532409E
5139A981 FED640A5
A14BEA42 583E40AD
DE6533F9 04F340B5
0C379F58 08A340BD
06DB1155 672A40C5
8481151F 248C40CE
9D6BCAD6 44FC40D7
94E1EC2A CCDE40E0
2439E7DD COC640EA
86CC1524 257D40F5
000000000 00004100
                                                                                                                                                                                                                                                                                                                                       *X00000000000004080
*X487B67CCAAC34085
*X8BD7E3EA95C1408B
*X11C373ABC3D34091
*XB8A9518D37F04098
*XA1126091F532409E
*X5139A9B1FED640A5
*XA14BEA42583E40AD
*XDE6533F904F340B5
*X0C379F5808A340BD
*X06DB1155672A40C5
*X8481151F248C40CE
*X9D6BCAD644FC40D7
*X94E1EC2ACCDE40E0
*X2439E7DDC0C640EA
*X86CC1524257D40F5
*X00000000000004100
                                                                                                                                                                                                                                    A1_TABLE:
                                                                                                                                                                                                                                                                                        QUAD.
                                                                                                                                                                                                                                                                                         .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                        QUAD.
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                         .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                        .QUAD
                                                                                                                                                                                                                                                                                         .QUAD
                                                                                                                                                                                                                                                                                         .QUAD
                                                                                                                                                                                                                                                                                         .QUAD
```

VAX

Pse

Cro

The 122 The 741

Mac _\$2

88

The

MAC

```
0108
0108
0118
0118
0118
0128
0138
0148
0168
0178
0178
0180
                                                                             1745
1776
1776
1777
1789
1812
1831
1845
1847
1849
1901
                                                                                       A2_TABLE:
00000000 00000000
0AA6DC61 2E4A2326
3AD6EBC5 20DCA348
B96B6382 3F5B23D8
5C864630 8D5A245E
288CC1EC BEDDA424
477C04EF 1A14A32F
29975CDC D9FDA3E6
156BEC99 4D04A477
25E1C093 AEFDA402
6C8ABCDA 0754A38B
506AE383 EE53A360
FD2C4466 6597A2BC
6E9AA824 32C42308
C1B9D6CD 40B523BB
B66031EB EE7423B1
00000000 00000000
                                                                                                                                      .QUAD
                                                                                                                 .QUAD
                                                         0188
                                                                                                                .QUAD
                                                         0190
                                                                            194 TWO_M55:.QUAD
195
196 C: .QUAD
197 C1: .QUAD
 00000000 00002500
                                                         0190
                                                                                                                                       ^x0000000000002500
                                                         0198
                                                                            196 C:
197 C1:
198 C2:
17F1295C AA3B44B8
00002000 AA3B44B8
E8800BBB C17F3695
                                                                                                                                       ^X17F1295CAA3B44B8
^X00002000AA3B44B8
                                                         0198
                                                                                                                QUAD.
                                                        01A0
                                                        01A8
                                                                                                                .QUAD
                                                                                                                                        ^XE8800BBBC17F3695
                                                         01B0
                                                                             199
                                                                            199
200 LOGTAB:
201
202
203
204
205 LOGLEN =
206
207
208 EXPTAB:
209
210
211
212
213
214
215
216
217 EXPLEN =
973BB7EA 01121CC2
2742C3D0 CA802581
F145FD62 19A02E3D
1DA4FD64 FE9F3723
00000000 00000000
00000005
                                                                                                                                       ^X973BB7EA01121CC2
^X2742C3D0CA802581
^XF145FD6219A02E3D
^X1DA4FD64FE9F3723
                                                         01B0
                                                                                        LOGTAB: .QUAD
                                                        01B8
                                                                                                                .QUAD
                                                       01C0
01C8
01D0
01D8
                                                                                                                .QUAD
                                                                                                                .QUAD
                                                                                                                                       ^X00000000000000000
                                                                                                                .QUAD
                                                                                        LOGLEN = <.-LOGTAB>/8
                                                         0108
                                                        0108
                                                        01D8
01D8
OD2F11A3 E6F41FFF
2D1FB089 85242521
6DC43BFA C3FF2A2E
D9857D84 955B2F1D
0627B825 584633E3
2C9CFC16 FDEF3875
CF7AF7D1 72173CB1
000000000 00000000
000000008
                                                                                                                                      ^X0D2F11A3E6F41FFF
^X2D1FB08985242521
^X6DC43BFAC3FF2A2E
^XD9857D84955B2F1D
^X0627B825584633E3
^X2C9CFC16FDEF3875
^XCF7AF7D172173CB1
                                                                                                                                                                                                                       0.27094695278739704E-19
0.35024086731698588E-16
0.38805761563274748E-13
0.35830323049894278E-10
0.26466421444330986E-07
0.14662262387640433E-04
0.54152123481245728E-02
                                                                                                                .QUAD
                                                        01E0
                                                                                                                .QUAD
                                                        01E8
01F0
01F8
0200
0208
0210
0218
0218
                                                                                                                .QUAD
                                                                                                                .QUAD
                                                                                                                .QUAD
                                                                                                                .QUAD
                                                                                                                .QUAD
                                                                                                                                       ^x000000000000000
                                                                                                                .QUAD
                                                                                        EXPLEN = <.-EXPTAB>/8
```

OTS Tat

.SBTTL OTS\$POWDD - DOUBLE to DOUBLE giving DOUBLE result

FUNCTIONAL DESCRIPTION:

OTS\$POWDR - DOUBLE result = DOUBLE base ** FLOATING exponent OTS\$POWDD - DOUBLE result = FLOATING base ** DOUBLE exponent OTS\$POWDD - DOUBLE result = DOUBLE base ** DOUBLE exponent

The DOUBLE result is given by:

base	exponent	result
= 0 = 0 = 0	> 0 = 0 < 0	0.0 Undefined Exponentiation Undefined Exponentiation
< 0	any	Undefined Exponentiation
> 0 > 0 > 0	> 0 = 0 < 0	2*(exp * LOG2(base)) 1.0 2*(exp * LOG2(base))

Floating Overflow and Underflow can occur.

Undefined Exponentiation can occur if:

1) base is 0 and exponent is 0 or negative

base is negative

The basic approach to computing x**y as 2^[y*log2(x)] is the following:

was computed.

Step 3: Evaluate 2^[y*log2(x)] accurate to the precision of the datatype in question.

To determine the accuracy to which log2(x) must be computed to, write y*log2(x) as I+h, where I is the integer closest to y*log2(x), and h=y*log2(x)-I (Note that lh!=<1/2.) Then

$$2^{(y+\log 2(x))} = 2^{(1+h)} = (2^1)^{(2^h)}$$
.

Since the factor 2°I can be incorporated into the final result by an integer addition to the exponent field, we can assume that the multiplication by 2°I incurs no error. Thus the total error in the final result is determined by how accurately 2°h can be computed. If the final result has p fraction bits, we would like h to have at least p good bits. In fact it would be nice if h had a few extra guard bits, say 4. Consequently, we would like h to be accurate to p + 4 bits.

Let e be the number of bits allocated to the exponent field of the data type in question. If I requires more that e bits to represent, then overflow or underflow will occur. Therefore if the product y*log2(x) has e+p+4 good bits, the final result will be accurate. This requires that log2(x) be

computed to at least p + e + 4 bits.

Since log2(x) must be computed to more bits of precision than is available in the base data type, either the next level of precision or multi-precision arithmetic must be used. We begin by writing

$$log2(x) = log2(b) +$$
 $c(2n+1)*z^{2}$,
 $n=0^{-}$

Where c(1) = 1, and $z' = (2/\ln 2)[(z-b)/(z+b)]$. Hence

$$log2(x) = log2(b) + z' + > c(2n+1)*z'$$

$$= log2(b) + z' + p(z').$$

Note that if p(z') is computed to p bits, and log2(b) + z' is computed to p+e+4 bits and overhangs p(z') by e+4 bits, the required accuracy will be achieved. Consequently, the essential tricks, are to pick b such that the overhang can be achieved and to compute log2(b) + z' to p + e + 4 bits.

CALLING SEQUENCE:

power.wd.v = OTS\$POWDR (base.rd.v, exponent.rf.v) power.wd.v = OTS\$POWRD (base.rf.v, exponent.rd.v) power.wd.v = OTS\$POWDD (base.rd.v, exponent.rd.v)

INPUT PARAMETERS:

Base and exponent parameters are call by value

IMPLICIT INPUTS: none

OUTPUT PARAMETERS: none

IMPLICIT OUTPUTS: none

FUNCTIONAL VALUE:

OTS\$POWDR - DOUBLE base ** FLOATING power
OTS\$POWRD - FLOATING base ** DOUBLE power
OTS\$POWDD - DOUBLE base ** DOUBLE power

SIDE EFFECTS:

SIGNALS MTH\$K_FLOOVEMAT if floating overflow.
SIGNALS MTH\$K_FLOUNDMAT if floating underflow.
SIGNALS MTH\$ UNDEXP (82 = 'UNDEFINED EXPONENTIATION') if
1) base is 0 and exponent is 0 or negative

0TS\$POWDD 2-007

- DOUBLE PRECISION ** DOUBLE PRECISION p 16-SEP-1984 01:57:20 VAX/VMS Macro V04-00 Page 8 OTS\$POWDD - DOUBLE to DOUBLE giving DOUB 6-SEP-1984 11:28:03 [MTHRTL.SRC]OTSPOWDD.MAR;1 (4)

01:

218 335 218 336 2) base is negative

```
O7FC
                                                       .ENTRY OTSSPOWDD, ACMASK
                                               Move x to RO/R1. If x < 0, or x = 0 and y = < 0, return 'UNDEFINED EXPONENTIATION' error condition, otherwise attempt to compute x**y
                                            GET_BASE:
              04 AC
                         70
                                                       MOVD
                                                                 base(AP), RO
                                                                                                : R0/R1 <-- x
                                            COMMON:
                  18
06
AC
01
                         14
19
73
15
                                                       BGTR
                                                                 DEFINED
                                                                                                 ; If x > 0 attempt to compute x**y
                                                                 UNDEFINED
                                                                                                   Branch to error code for x < 0 (heck sign of y (Note that x = 0)
                                                       BLSS
              00
                                                       TSTD
                                                                 exp(AP)
                                                                 UNDEFINED
                                                       BLEQ
                                                                                                   Branch to error condition if y =< 0
                                               If processing continues here, this implies that x = 0 and y > 0. Return
                                               x**y = 0
                                                       RET
                                                                                                 : Return
                                               If processing continues here, this implies that an undefined exponentiation was attempted. Signal error and return
                                       UNDEFINED:
                        3C
D4
9A
FB
           8000 8F
                                                       MOVZWL
                                                                 #*X8000, RO
                                                                                                : RO/R1 <-- Reserved operand
                                                       CLRL
7E 00000000 GF
                                                       MOVZBL
                                                                 #MTH$K_UNDEXP, -(SP)
                                                                                                   Put error code on stack
                                                                 #1. GAMTHSSSIGNAL
                                                                                                   Convert error number to 32 bit
                                                       CALLS
                                                                                                      condition code and signal error.
                                                                                                      NOTE: Second argument is not re-
                                                                                                      quired since there is no JSB entry.
                                                                                                   Return
                                                       RET
                                       396
397
398
399
                                               If processing continues here will attempt to compute x**y as 2^{(y*log2(x))}.
                                               We begin by determining k and f such that x = 2^k \cdot f, where 1 = 0?
                                       400
401
402
403
404
405
406
407
408
409
410
                                            DEFINED:
                                                                                                R4 <-- 2*7*(biased exponent of x)
R4 <-- 2*7*k = 2*7*(exponent of x - 1)
R0 <-- f = 2*(fraction field of x)
      FFFF807F 8F
00004080 8F
50 54
                                                       BICL3
                                                                 #^XFFFF807F, RO, R4
                                                       SUBL
                                                                 #*X4080, R4
                                                                 R4. R0
                                                       SUBL
                                               We are now ready to compute log2(x). This computation is based on the
                                               following identity:
                                                   log2(2^k*f) = k + log2(a) +
                                                                                                      ---- z^{(2j+1)}, where z =
                                               We begin by determining a as b^{-1}, where b = 2^{-1/16} and i is between 0 and 16 inclusive. Specifically i is chosen by table look-up
```

: R4 <-- high longword of f + al

```
in such a fashion as to minimize the magnitude of z. Since log2(a) = i/16
                                                         we may write
                                                41890122342267890
442234424267890
                                                                                log2(x) = k + 1/16 + z*p(z*z).
                                                      EVAL_LOG2:
SA
                                 CB
90
C1
90
       50
            FFFFFF80 8F
                                                                 BICL3
                                                                                                               R10 <-- index to INDEX table R10 <-- i*2^3
                FD72 CF4A
                                                                            INDEX[R10], R10
                                                                 MOVB
                                                                            R10, R4, -(SP)
#-3, R10, R10
                                                                 ADDL3
                                                                                                                           2^7*(k + 1/16)
                                                                                                                SP -->
                   FD 8F
               5A
                                                                                                                R10 <--
                                                                 ROTL
                                                                                                                R10 will be multiplied by 2°3 by
                                                                                                                 table references like the line below.
                                                                                                                 The linker will cause an error if
                                                                                                                 () are used instead of [] for these
                                                                                                                 table references.
                                                        We proceed by computing z = (f-a)/(f+a). In order to insure the accuracy of
                                                        the final result, it is necessary to compute z to at least 68 bits. Since no back up data type is available, we must compute z in two parts: z=z1+z2, where z1 is the high 24 bits of z and z2 is the low 56 bits of z. Further, to obtain the desired accuracy it is necessary to work with a=a1+a2, where a1 and a2 are the high and low 56 bits respectively of 'a'. We begin
                                                        computing (in single precision)
                                                                                     z1 = (f - a1)/(f + a1)
                                                        Note that f-al can be computed exactly in 56 bits, but f+al may require 57
                                                        bits. The 57 bit can be determined by the exclusive or of the low bits of
                                                        f and al.
        58<sub>7E</sub>
               FDE3 CF4A
51 59
                                 7D
CD
                                                                           A1_TABLE[R10], R8
R9, R1, -(SP)
                                                                 MOVQ
                                                                                                               R8/R9 <-- a1
                                                                                                                SP --> XOR of low bits of al and x
                                                450
451
453
454
455
456
457
458
                                                                 XORL3
                                                                                                                     (This will used to determine the
                                                                                                                       57 bit of f+a1.)
                                                                           R8.
R8.
R2.
                                 61
62
47
D0
                   50
50
50
59
                          58
58
52
00
                                                                                                               R2/R3 <-- f + a1 (rounded)
R0/R1 <-- f - a1 (exact)
                                                                 ADDD3
                                                                                 RO
RO,
                                                                 SUBD
            58
                                                                 DIVF3
                                                                                                               R8 <-- z1 (single)
                                                                 MOVL
                                                                                                               R8/R9 <-- z1 (double)
                                      02B0
                                                        To compute 22 we note
                                                460
                                                               z = z1 + z2 = (f - a1 - a2)/(f + a1 + a2)
                                                461
                                                                           z2 = (f - a1 - a2)/(f + a1 + a2) - z1
                                                464
                                                        Now let v = f + a1 + a2 = v1 + v2, where v1 and v2 are the high 24 and low
                                                        56 bits of v respectively. Then
                                                466
                                                                            z^2 = [(f - a^1 - z^{1*v^1}) - (a^2 + z^{1*v^2})]/v
                                                468
469
470
                                                        We begin by computing v1 and f - a1 - z1*v1
                   54
                          52
                                 DO
                                                                 MOVL
                                                                            R2. R4
```

```
- DOUBLE PRECISION ** DOUBLE PRECISION p 16-SEP-1984 01:57:20 VAX/VMS Macro V04-00 OTS$POWDD - DOUBLE to DOUBLE giving DOUB 6-SEP-1984 11:28:03 [MTHRTL.SRC]OTSPOWDD.MAR;1
                                                                                                                                          : R4/R5 <-- v1
: R6/R7 <-- f + a1 - v1 (exact)
: R4/R5 <-- z1*v1 (exact)
: R0/R1 <-- f - a1 - z1*v1 (exact)
                                                                                             #0, R5
R4, R2, R6
R8, R4
R4, R0
                                    00 63 62
         56
                                                                               SUBD3
                                                                               MULD
                                                                               SUBD
                                                                    Compute v2 and a2 + a1*v2
          FFFEFFFF 8F
     54
                                     7D CA 13 60 60 60 60
                                                                                              AZ TABLE[R10], R4
                                                                                                                                             R4/R5 <-- a2
                                                                               BICL
                                                                                                                                              Check if w was rounded
                                                                               BEQL
                                                                                                                                              Branch if not rounded
                                                                                             TWO_M55, R6
R4, R6
R8, R6
R4, R6
                            CF
54
58
54
                  FEBD
         56
                                                                               SUBD
                                                                                                                                              Correct for rounding error (exact)
                                                                                                                                             R6/R7 <-- v2
R6/R7 <-- z1*v2
R6/R7 <-- a2 + z1*v2
                  56
56
56
                                                                               ADDD
                                                                               MULD
                                                                               ADDD
                                                                    Compute z2
                  50
                           56
52
                                     62
                                                                                                                                          : RO/R1 <-- (f-a1-z1+v1)-(a2-z1+v2)
: RO/R1 <-- z2
                                                                               SUBD
                                                                               DIVD
                                                                    The next step is to compute log2(x) accurate to at least 68 bits. This is
                                                                    accomplished as follows, let
                                                                                           w = 2^7*log2(x)

= (2^7)[k + i/16 + z*p(z*z)]

= 2^7*(k + i/16) + (2^7)*z*(c0 + c2*z^2 + ... + c10*z^10)

= [2^7*(k + i/16) + z'] + z'(d2*z'^2 + ... + d10*z'^10)

= [2^7*(k + i/16) + z'] + z'*q(z'*z')

= w1 + w2
                                                                   where z'=(2^*7*c0)*z and w1 and w2 are the high 24 and low 56 bits of w respectively. Note that the choice of 'a' used in computing z, guarantees that z' overhangs z'*q(z'*z') by at least 13 bits. Hence, if w is computed as w1 + w2, the necessary 68 bits of accuracy can be obtained. c0=2^*6/\ln(2).
                                                                    We begin by defining
                                                                                             c = high 56 bits of (2*7*c0)
c1 = high 28 bits of (2*7*c0)
c2 = low 56 bits of (2*7*c0)
                                                                    then
                                                                                             z' = (z1 + z2)*(c1 + c2)
= z1*c1 + z1*c2 + z2*c.
                                                                                             C1, R8, R4
C2, R8
C, R0
R8, R0
         58
58
50
54
                                                                                                                                              R4/R5 <-- c1*z1
                                     65
64
64
60
61
                                                                               MULD3
                                                                                                                                            R8/R9 <-- c2*z1
R0/R1 <-- c*z2
R0/R1 <-- c*z2 + c2*z1
                  FEBB
FEA6
50
50
                           CF
CF
58
54
                                                                               MULD
                                                                               MULD
                                                                               ADDD
         58
                                                                               ADDD3
                                                                                              R4, RO, R8
                                                                                                                                             R8/R9 <-- z'
                                                                    We proceed by letting
                                                                                              w1 = high 24 bits of 2^7*(k + i/16) + z1*c1
                                                                    and
                                                                                              w2' = \{[2^7*(k + i/16) + z1*c1 - w1] + z1*c2\} + z2*c.
```

OTSSPONDD 2-007

```
- DOUBLE PRECISION ** DOUBLE PRECISION p 16-SEP-1984 01:57:20 DTS$POWDD - DOUBLE to DOUBLE giving DOUB 6-SEP-1984 11:28:03
                                                                                                                                 VAX/VMS Macro V04-00
[MTHRTL.SRC]OTSPOWDD.MAR:1
                                                                               2^7*(k + 1/16) + z' = w1 + w2'
                                                                                   = [2^7*(k + i/16) + z'] + z'*q(z'*z')
= w1 + w2' + z'*q(z'*z')
= w1 + w2,
                                                           where w2 = w2' + z'*q(z'*z')
                  04 AE
5 5A
7 00
6 5A
                                                                                4(SP), R10
R10, R4, R6
#0, R7
R10, R6, R2
                               4E
41
00
43
                                                                                                                         R10 <-- 2^7(k + 1/16)
R6 <-- 2^7(k+1/16) + z1*c1 in single
           5A
                                                                    CVTLF
                                                                    ADDF 3
                                                                                                                          R6/R7 <-- w1
                                                                    MOVL
                                                                                                                          R2 <-- bits of z1*c1 included in w1 = -[2(k+i/16)+z1*c1 - w1]
                                                                    SUBF 3
                        00
52
54
                                D0
62
61
                                                                                #0, R3
R2, R4
R4, R0, (SP)
                                                                                                                         Convert R2 to double
[2^7(k+i/16)+z1*c1 - w1] + c1*z1
(SP) --> w2'
                                                                    MOVL
                                                                    SUBD
                                                                    ADDD3
                                                       Compute w2
                                                                                R8, R8, R0
R0, #LOGLEN-1, LOGTAB
                                                                                                                      : R0/R1 <-- z'*z'

: R0/R1 <-- q(z'*z')

: R0/R1 <-- z'*q(z'*z')
                        58
50
58
6E
FE93 CF
                                65
75
64
60
                                                                    MULD3
                                                                    POLYD
                                                                                 R8, R0
(SP), R0
                                                                    MULD
                                                                                                                       : RO/R1 <-- w2
                                                                    ADDD
                                                          We now calculate y*log2(x) = (y1+y2)*(w1+w2) = y1*w1 + y2*w1 + y*w2, where y1 and y2 are the high and low 28 bits of y respectively.
                  OC AC
                               70
                                                                    MOVQ
                                                                                 exp(AP), R4
                                                                                                                       : R4/R5 <-- y
                                                          Test for the possibility of overflow in the computation of y*w1.
                                                           This will occur if the exponent of y plus the exponent of wi is greater
                                                           than 127.
                                                                                #7, #8, R4, R2
#2, #8, R6, R3
#2, #8, R6, R3
#2, R3
#2, R3
#27F, R3
Y TIMES_W1_OVER
                080
0080
08
0080
52
                                                                    SUBW2
                                                                                                                         biased exp of y unbiased exp of y
                               EAF201940B244
53
                                                                    EXTZV
                                                                                                                          biased exp of w1
                                                                    SUBWA
                                                                                                                          unbiased exp of w1
               53
007F
                                                                    ADDW2
                                                                                                                          unbiased exp of w1*y
                                                                    CMPW
        53
                                                                                                                          largest unbiased exp possible is 127
                                                                                R4 R2
                                                                                                                         RO/R1 <-- y*w2
R2 <-- high longword of y1
R2/R3 <-- y1
R4/R5 <-- y2
R2/R3 <-- y1*w1
R4/R5 <-- y2*w1
                                                                    MULD
                                                                    MOVL
         FFFFOFFF
 55
                                                                    BICL3
                                                                                 #*XFFFFOFFF, R5, R3
                                                                                R2. R4
R6. R2
R6. R4
                                                                    SUBD
                                                                    MULD
                                                                    MULD
                                                       The next step in computing 2*[y*log2(x)] is to write y*log2(x) as
```

OTS Sym

UND

PSE ---SAB OT

> Pha ---Ini Com Pas

Sym Sym Cro

#1, G^MTH\$\$SIGNAL

CALLS

.END

03DA

03DA

03DA

03E1

03E2 03E2

FB 04

01

00000000 GF

VAX ASS The 916 The 380

OTS

Mac ---

_\$2 148

The MAC

so can be fixed up by any error

return - RO restored from CHF\$L_MCH_RO/R1

handler

signal condition

```
- DOUBLE PRECISION ** DOUBLE PRECISION p 16-SEP-1984 01:57:20 VAX/VMS Macro V04-00 6-SEP-1984 11:28:03 [MTHRTL.SRC]OTSPOWDD.MAR;1
OTSSPOWDD
                                                                                                                                                                                    Page
Symbol table
A1_TABLE
A2_TABLE
ACMASK
                         00000080 R
00000108 R
                      = 000007FC
= 00000004
BASE
                         00000198
                                               00000148
000001A8
00000252
0000026F
00000281
COMMON
DEFINED
EVAL_LOG2
EXCEPTION_1
                         000003B1 R
EXCEPTION_2
                      000003B7 R
= 0000000C
EXPLEN
                      = 00000008
EXPTAB
                         000001D8 R
                                               02
                      = 00000008
FEXP
                         0000024E R
GET_BASE
INDEX
LOGLEN
                      = 00000005
                         000001B0 R
LOGTAB
                                               MTH$$SIGNAL
                         *******
MTHSK_FLOOVEMAT
MTHSK_FLOUNDMAT
MTHSK_UNDEXP
OTSSPOUDD
                         *******
                         *******
                         *******
                        0000024C RG
00000218 RG
00000232 RG
000003D2 R
000003B0 R
00000004
00000190 R
0000025C R
000003BF R
OTS$POWDR
OTS$POWRD
OVER
RETURN
SFSW_SAVE_PSW
TWO_MSS
                                               02
02
02
02
UNDEFINED
UNDER
Y_TIMES_W1_OVER
                         000003BB R
                                                                      +-----
                                                                         Psect synopsis
PSECT name
                                               Allocation
                                                                            PSECT No.
                                                                                            Attributes
                                                                            00 ( 0.)
01 ( 1.)
02 ( 2.)
                                               00000000
                                                                                                                                                                  NOWRT NOVEC BYTE
    ABS
                                                                                            NOPIC
                                                                                                        USR
                                                                                                                                  LCL NOSHR NOEXE NORD
$ABS$
                                                                    0.)
                                                                                            NOPIC
                                               00000000
                                                                                                        USR
                                                                                                                 CON
                                                                                                                         ABS
                                                                                                                                  LCL NOSHR
                                                                                                                                                   EXE
                                                                                                                                                            RD
 OTS$CODE
                                               000003E2
                                                                                                        USR
                                                                                                                 CON
                                                                                                                                           SHR
                                                                                                                                                            RD
                                                                                                                                                                  NOWRT NOVEC QUAD
                                                                                                                                                    EXE
                                                                  ! Performance indicators !
                                                           CPU Time
Phase
                                     Page faults
                                                                                 Elapsed Time
                                                           -----
                                                          00:00:00.10
00:00:00.75
00:00:02.35
00:00:00.05
00:00:01.51
00:00:00.06
                                                                                00:00:00.63
00:00:06.82
00:00:06.76
00:00:00.10
00:00:04.55
00:00:00.54
                                               32
125
134
Initialization
Command processing
Pass 1
Symbol table sort
                                               130
Pass 2
Symbol table output
```

**

OTS

```
OTS$POWDD - DOUBLE PRECISION ** DOUBLE PRECISION p 16-SEP-1984 01:57:20 VAX/VMS Macro V04-00 Page 17 (7)
Psect synopsis output 1 00:00:00.02 00:00:00.02
```

Cross-reference output 0 00:00:00.00 00:00:00.00 Assembler run totals 428 00:00:04.85 00:00:19.43

The working set limit was 1200 pages.

The working set limit was 1200 pages.
12217 bytes (24 pages) of virtual memory were used to buffer the intermediate code.
There were 10 pages of symbol table space allocated to hold 62 non-local and 2 local symbols.
741 source lines were read in Pass 1, producing 21 object records in Pass 2.
9 pages of virtual memory were used to define 8 macros.

! Macro library statistics !

Macro library name

Macros defined

\$255\$DUA28:[SYSLIB]STARLET.MLB:2

4

88 GETS were required to define 4 macros.

There were no errors, warnings or information messages.

MACRO/ENABLE=SUPPRESSION/DISABLE=(GLOBAL, TRACEBACK)/LIS=LIS\$:OTSPOWDD/OBJ=OBJ\$:OTSPOWDD MSRC\$:MTHJACKET/UPDATE=(ENH\$:MTHJACKET)+MSRC

0264 AH-BT13A-SE

DIGITAL EQUIPMENT CORPORATION CONFIDENTIAL AND PROPRIETARY

